Qualifying exam - January 2011

Statistical Mechanics

NAME:______________________________

You can use a graduate level textbook and a calculator. Please write legibly and show all steps of your derivations and/or calculations.

Problem 1 [20 points]

Consider a hypothetical substance for which

$$U = AVT^n \tag{1}$$

and

$$S = BVT^3, \tag{2}$$

where U is internal energy, V volume, S entropy, T temperature, and A and B are constant coefficients.

1. Find the exponent n and the ratio A/B.
2. Find the pressure in the substance.
3. Find the chemical potential of the substance.

Problem 2 [20 points]

Calculate the internal energy (in J/mole) and specific heat at a constant volume (in J/mole/K) of carbon dioxide CO_2 at the temperature of 1000 K. Consider CO_2 as an ideal gas and treat the molecular rotations and vibrations in the classical limit. The CO_2 molecule has a linear structure O=C=O. (The gas constant is $R = 8.314$ J/mole/K.)

Problem 3 [30 points]

A paramagnetic salt can be modeled as a set of fixed non-interacting magnetic moments μ oriented either parallel or antiparallel to an applied magnetic field. Suppose the kinetic energy of the ions carrying the magnetic moments can be neglected.

1. The salt is equilibrated with a thermostat at a temperature T and the magnetic field is slowly increased from zero to B at the fixed temperature. What is the amount of heat released by one mole of the salt during this process?

2. The salt is now thermally isolated and the magnetic field is adiabatically decreased from B to $B/10$. What is the final temperature of the salt?

(This process of adiabatic demagnetization is used for cooling materials to very low temperatures. The real process is more complex than assumed in this problem.)

Problem 4 [30 points]

1. In the Einstein model, a solid containing N atoms is represented by a set of $3N$ identical but distinguishable harmonic oscillators of frequency ν. The chemical bonding between the atoms lowers the free energy by εN. Assume that ν and ε are independent of the volume. Present a complete derivation of the chemical potential of atoms in this solid.

2. If the solid evaporates, it forms an ideal atomic gas. Present a complete derivation of the chemical potential in the vapor.

3. Assuming that the vapor is in thermodynamic equilibrium with the Einstein solid (saturated vapor), show that the pressure in the vapor equals

$$p = kT \left(\frac{2\pi mkT}{h^2} \right)^{3/2} \left(2 \sinh \frac{h\nu}{kT} \right)^3 e^{-\varepsilon/kT}, \tag{3}$$

   where m is the atomic mass.