Classical Mechanics Qualifying Exam, January 2017

You have three hours to complete the exam and may use your graduate textbook.

1. [30] A particle of mass m is attached to an ideal, massless spring with spring constant k and oscillates along the x-axis with amplitude A_0. The equilibrium position is at $x = 0$ and the other end of the spring is held fixed at a position with $x < 0$. At time $t = 0$, the particle is located at $x = A_0/2$ and is moving in the positive x-direction. At that moment, it collides with another particle of mass m which was initially at rest. The collision occurs instantaneously (i.e. it has zero time duration) and the two particles stick together.

(a) [15] Find the speed of the combined particle of mass $2m$ immediately following the collision.
(b) [10] Find the position as a function of time, $x(t)$, for the combined particle after the collision.
(c) [5] Explain why the amplitude A of the oscillation following the collision is not the same as the initial amplitude A_0.

2. [35] An axially symmetric space station is floating in free space. The symmetry axis lies along $\mathbf{e}_3$. Due to the symmetry, its principal moments of inertia $(\lambda_1, \lambda_2, \lambda_3)$, corresponding to body axes $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, are such that $\lambda_1 = \lambda_2$. The space station has rockets mounted symmetrically on either side that are firing and exert a constant torque $\Gamma\mathbf{e}_3$. Find the rotational velocity $\boldsymbol{\omega}$ of the station (relative to the body axes) as a function of time and describe the motion. At time $t = 0$, take $\boldsymbol{\omega} = (\omega_{10}, 0, \omega_{30})$.

3. [35] Consider two plane pendulums that are joined by an ideal, massless spring with spring constant k (see figure). Each consists of a massless string

of length L attached to a small bob of mass m (pendulum on the left) or $2m$ (pendulum on the right). The orientations of the pendulums are specified by the angles ϕ_1 and ϕ_2 as shown. The natural length of the spring is equal to the distance between the two supports, so the equilibrium position is at $\phi_1 = \phi_2 = 0$ with the two pendulums vertical. Assuming $\phi_1 \ll 1$ and $\phi_2 \ll 1$ at all times, find the normal modes for this system.