NAME:

QM, Qualifying Exam: 2010

Note: This is an open book exam and you are allowed to bring " Principles of Quantum Mechanics" by Shankar. If a formula appears in the Shankar's book, please use that as a starting point, there is no need to show the derivation of that formula.

(1) (20 points)

Let ψ_1, ψ_{-1} and ψ_0 are the eigenstates of the L_z operator with eigenvalues $1, -1, 0$ respectively. Consider a particle in a normalized state described by,

$$\psi = \frac{1}{2}e^{i\theta_1}\psi_1 + \frac{1}{2}e^{i\theta_{-1}}\psi_{-1} + \frac{1}{\sqrt{2}}e^{i\theta_0}\psi_0$$

where θ_i, $(i = 1, -1, 0)$ are real constants.

(a) Calculate the probability of finding the particle in a state with $L_x = 0$.

(b) Calculate the probability of finding the particle in a state with $L_z = 0$.

(c) If at $t = 0$, measurement of L_z gives 1, what will the measurement of L_z give at a later time ??

(2) (20 points)

Consider a system of three non-interacting particles (each of unit mass) confined in a two-dimensional square box of unit dimensions. Calculate the total ground state energy and the first excited state energy of the system if, (a) Particles are non-interacting neutrons. (b) Particles are non-interacting He^4 atoms. (c) Two particles are electrons and the other is a positron.

(d)Consider a composite object such as the H-atom. Will it behave as a boson or fermion??

(3) (10 points)
Use uncertainty principle to obtain ground state energy of a particle of unit mass in a one-dimensional box of length unity. Compare the resulting answer with the actual value of the ground state energy of the system.

(4) (20 points)
A particle is moving in a harmonic potential,
$V(x) = 3(x^2 + y^2 + z^2)$. Solve the problem in two different coordinate systems and write down the ground and the first excited state wave functions in these two coordinates. Are the solutions degenerate?? If yes, what is the degeneracy of the ground and the first excited states ??

(5) (10 points)
If a particle is described by a wave function, $\psi(x, y, z) = Ae^{-2r^2}(x+z)$ where A is a normalization constant. Calculate the possible angular momentum quantum numbers of the system and the probability of being found in those states.

(6) (20 points)
Consider a free particle , with initial wave function given by the

wave packet,

$$\psi(x, t = 0) = Ae^{ix}e^{-(x-2)^2}$$

where A is a normalization constant and both x and t are dimensionless. (a)What is the mean position and mean momentum of the particle at $t = 0$.??
(b)What is the mean position and mean momentum of the particle at $t = 5$.
(c)What is the wave function of the particle at $t = 5$??
(d) How does the answers to (a)-(c) change if $\psi(x, t = 0)$ describes the ground state wave function of a harmonic oscillator.