George Mason University

Physics PhD Qualifying Exam — 8 December 08

Instructions: Answer all the questions. Writing must be legible to get credit.

1. The Hamiltonian operator describing a certain quantum mechanical system has a matrix representation:

$$H = E_0 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

H has eigenvalues $2E_0$ and 0.

Another operator, A, has the representation

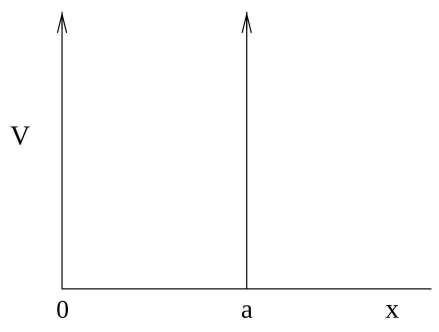
$$A = a \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$

A has eigenvalues $(1+\sqrt{2})a$ and $(1-\sqrt{2})a$.

   (a) Find the eigenvectors of H and A in this representation.

   (b) Suppose that a measurement of the energy of the system is made, and the result is $2E_0$. The quantity A is then measured. Calculate the probabilities of the results $(1+\sqrt{2})a$ and $(1-\sqrt{2})a$.

2. A particle of mass m is in a 1-dimensional infinite square well, of total width a, as



shown in the figure. At time $t = 0$ its normalized wave function is:

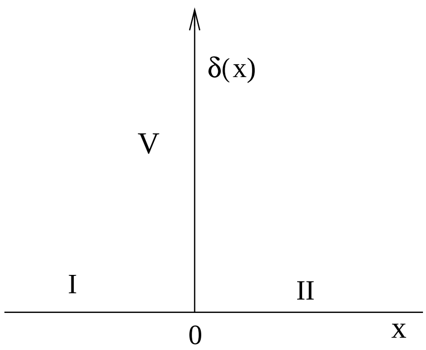
$$\psi(x, t = 0) = \sqrt{8/5a}[1 + \cos(\pi x/a)] \sin(\pi x/a)$$

   (a) Which energy levels are occupied for this wave function?

   (b) What is the wave function at some later time, t?

   (c) What is the average energy at $t = 0$, and later at t?

3.



A particle represented by a plane wave is incident, from the left, on a 1-dimensional potential barrier:

$$V = V_0 \delta(x)$$

Calculate the probability that the particle is reflected from the barrier. Use the continuity of the wave function at $x = 0$ and following relation for the discontinuity in the derivative of the wave function, at a δ-function:

$$\Delta(\frac{d\psi}{dx}) = \frac{2mV_0}{\hbar^2}\psi(0)$$

4. A spinless particle is represented by a wave function

$$\psi(x, y, z) = Kxe^{-\alpha r}$$

where K and α are real constants and $r = \sqrt{x^2 + y^2 + z^2}$ By writing x in terms of spherical harmonics,

(a) What is the total angular momentum of the particle?

(b) What is the expectation value of L_z?

(c) What is the probability that the particle's θ is less than 45°?

Some spherical harmonics:

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \qquad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}$$