Classical Electrodynamics Qualifying Exam: August 25, 2009

1. [15] Consider the electrostatic potential Φ on the x-y plane in the region $y > 0$, with $\Phi(x, y = 0) = V \cos(2\pi x/L)$ and $\Phi \to 0$ as $y \to \infty$. Find $\Phi(x, y)$ when $y > 0$.

2. [15] A point electric dipole with moment $p\,\hat{z}$ is located at the origin. A grounded, conducting, spherical shell has radius a and is centered on the origin. Find the electrostatic potential Φ for $r < a$.

3. [10] A sphere with radius a and made of a linear dielectric material with dielectric constant ϵ/ϵ_0 is placed in a region where there is initially a uniform electric field $E_0\,\hat{z}$. As a result, the potential inside the sphere becomes (in spherical coordinates)

$$\Phi(r, \theta) = -\frac{3\epsilon_0}{\epsilon + 2\epsilon_0} E_0 r \cos\theta$$

Find the bound charge volume density inside the sphere and the bound charge surface density on its surface.

4. [20] A circular loop with radius a is centered on the origin, lies in the x-y plane, and carries a current I.
a) [5] What is the magnetic dipole moment $\vec{m}$ of the loop?
b) [15] Find the magnetic vector potential $\vec{A}(\vec{x})$ in the limit that $|\vec{x}| \gg a$.

5. [20] Consider a long, straight wire with circular cross section. Each end is held at constant potential, with potential difference V. Although current may flow down the wire, the charge density vanishes throughout.
a) [10] Show that the electric field inside the wire is uniform.
b) [10] Use the Poynting vector to show that heat is dissipated in the wire at rate VI when current I flows.