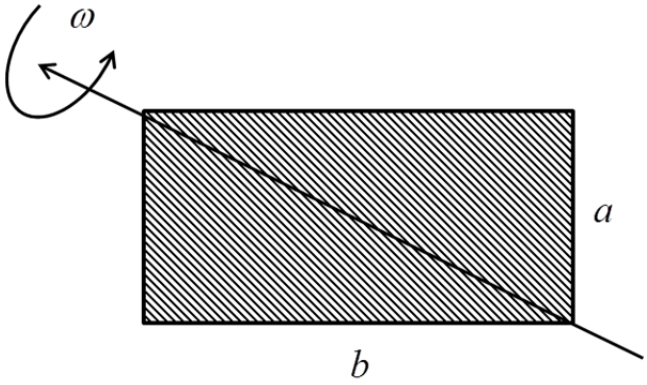
Classical Mechanics Qualifier (August 2014)
George Mason University

You will have **THREE** hours to complete the exam.
You are allowed to use your graduate textbook during the exam.

Problem 1 (20pts)



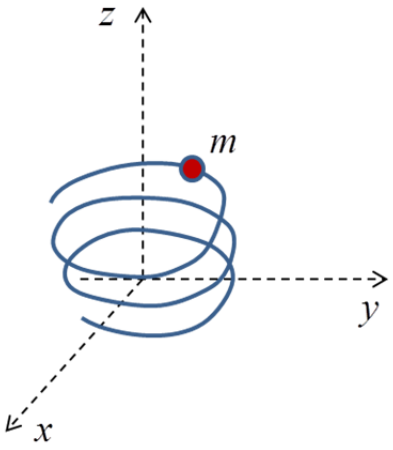
A uniform rectangular door with mass M, sides a and b ($b > a$) and negligible thickness rotates without the influence of gravity at a constant angular velocity ω about a diagonal through its center of mass as shown to the right.

i) What are the principle axes and moments of inertia for this door?
ii) What is the angular momentum vector in the body coordinate system?
iii) What external torque must be applied to keep this door to rotate at a constant angular velocity around this diagonal direction? Draw a picture showing vectoral relationship among the three vectors $\boldsymbol{\omega}, \mathbf{L}, \mathbf{N}$ in the body coordinate system.

Problem 2 (20pts)

A particle of mass m moves frictionlessly under the influence of gravity along a helical path: $z = k\theta$, $r = const$, where k is a constant and (r, θ) are the polar coordinates on the x-y plane.

i) Find the Lagrangian and the Hamiltonian for the particle.
ii) Determine the equation of motion using the Hamiltonian Equations.



Problem 3 (30pts)

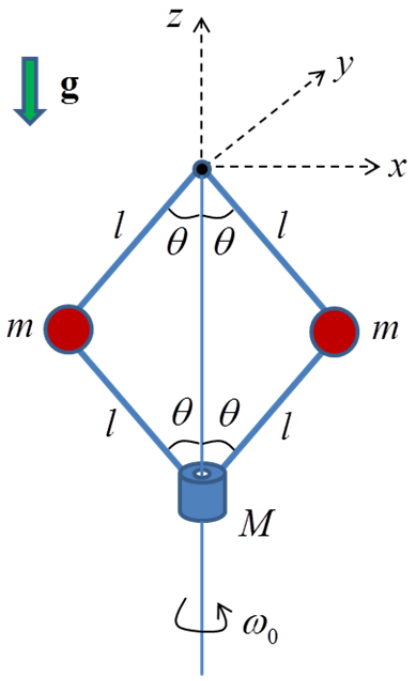
An old-style flyball governor for a steam engine consists of two masses m connected to arms of length l and another mass M as shown to the right. The whole assemble is constrained to rotate with a constant angular velocity ω_0 around a central shaft on which the mass M can slide up and down without friction. Neglect the mass of the arms, air friction and treat all masses as point masses.

i) Write down the Lagrangian for the system and derive its equation of motion in terms of the generalized coordinate θ.

ii) What are the two equilibrium values for θ?

iii) What is the minimum value for ω_0 so that θ will have a non-zero equilibrium value θ_0?

iv) If the system is perturbed around this non-zero equilibrium θ_0, what is the frequency of small oscillations around it?



Problem 4 (30pts)

A physical planar double pendulum is formed by two identical rods of length l and mass m connected to a horizontal platform and together by small flexible pieces of massless string. Find the frequencies of the normal modes of this system for small oscillations around the equilibrium position. Describe the motion of each of the normal modes.