Qualifying exam - August 2012

Statistical Mechanics

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [20 points]

1. [10 points] Two vessels are isolated and contain equal amounts of molecular oxygen O_2 at the same temperature T but different pressures p_1 and p_2. The vessels are then connected. Find the change in entropy and determine its sign.

2. [10 points] Two vessels are isolated and contain equal amounts of molecular oxygen O_2 at the same pressure p but different temperatures T_1 and T_2. The vessels are then connected. Find the change in entropy and determine its sign.

In this problem you can consider oxygen as an ideal gas and treat molecular rotations and vibrations in the classical approximation.

Problem 2 [32 points]

Consider an extreme relativistic gas of identical classical particles with the energy-momentum relation $\varepsilon = pc$, where c is the speed of light.

1. [8 points] Show that the partition function of the gas is

$$Z(V,T) = \frac{1}{N!}\left[8\pi V\left(\frac{kT}{hc}\right)^3\right]^N. \tag{1}$$

2. [8 points] Calculate the specific heats of this gas at constant volume (c_v) and constant pressure (c_p). Compare your results with the respective specific heats on a non-relativistic ideal gas of single atoms.

3. [8 points] Show that for the extreme relativistic gas $PV = E/3$, where P is pressure, V is volume and E is internal energy.

4. [8 points] If this gas expands reversibly and adiabatically from a volume V to a volume $3V$, what is the change in temperature?

Problem 3 [33 points]

According to the model proposed by Einstein (1907), a solid can be represented by a set of $3N_a$ identical but distinguishable quantum oscillators with the same frequency ω (N_a is the number of atoms in the solid). Suppose the vibration frequency depends on volume per atom, v, according to the relation

$$\omega = \omega_0 e^{-av}, \tag{2}$$

where ω_0 and a are constants.

1. [11 points] Calculate the isothermal compressibility, β_T, of the solid.
2. [11 points] Calculate its heat capacity, C_v, at a constant volume.
3. [11 points] Prove the following relation:

$$E = C_v T + \frac{N_a}{va^2 \beta_T}, \tag{3}$$

where E is the total energy of the solid and T is temperature.

Problem 4 [15 points]

Consider a three-dimensional free electron gas at zero temperature (degenerate electron gas). Calculate the relative root-mean-square deviation of its energy,

$$\frac{\left(\overline{(\varepsilon - \overline{\varepsilon})^2}\right)^{1/2}}{\overline{\varepsilon}}, \tag{4}$$

where ε is energy per electron.