Qualifying exam - January 2019

Statistical Mechanics

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [30 points]
Consider an ideal gas of classical particles of mass m, each carrying an electric dipole moment $\mathbf{P}$. The gas is contained in a box of volume V and is equilibrated with a thermostat at temperature T. A uniform electric field $\mathbf{E}$ is applied to the gas. Ignoring interactions between the particles, calculate

1. [6 points] Chemical potential as a function of temperature and pressure.
2. [6 points] Average energy $\bar{\varepsilon}$ per particle.
3. [6 points] The magnitude of the average dipole moment $\bar{P}$ per particle.
4. [6 points] The specific heat $(\partial\bar{\varepsilon}/\partial T)_E$ per particle.
5. [6 points] The dielectric susceptibility $\left(\partial\bar{P}/\partial E\right)_T$.

Problem 2 [30 points]
Consider a cavity containing black-body radiation at a temperature T_1. Suppose the volume of the cavity increases in an equilibrium adiabatic process from an initial value V_1 to a final value $V_2 = 5V_1$.

1. [5 points] What is the final temperature T_2 in the cavity?
2. [5 points] If the initial radiation pressure was p_1, what is the final pressure p_2?
3. [15 points] If the cavity initially contained a total of N_1 photons, what is the final number N_2 of photons in the cavity? Explain the physical meaning of this result.

Problem 3 [40 points]
Consider a quantum gas of ultra-relativistic particles (bosons or fermions) with the energy-momentum relation $\varepsilon = cp$, where c is the speed of light. Show that, regardless of temperature,

$$PV = \frac{E}{3}, \tag{1}$$

where P is pressure, V is volume of the gas and E is its total energy.

Does this result remain valid for an ultra-relativistic gas in the Maxwell-Boltzmann statistics?