Qualifying exam - August 2019

Statistical Mechanics

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [35 points]
A particular substance has the fundamental equation of state

$$S = AN \ln \frac{EV}{N^2}, \tag{1}$$

where S is entropy, E is energy, V is volume, N is the number of particles, and A is a constant. Calculate the following thermodynamic properties of the substance:

(a) Specific heat at constant volume.

(b) Specific heat at constant pressure p.

(c) Isothermal compressibility $\beta_T = -\left(\partial \ln V/\partial p\right)_{T,N}$.

(d) Chemical potential as a function of p and T.

(e) Imagine a planet with atmosphere formed by a gas with the fundamental equation (1). If the atmospheric pressure on the surface of the planet is p_0, what is the pressure at elevation H? The mass of a particle in the gas is m and the free fall acceleration on the surface of the planet is g. Assume that the temperature is independent of the elevation.

Problem 2 [40 points]
Derive an integral expression for the chemical potential (free energy per atom) of a Debye solid. Find the asymptotic values of this chemical potential in the limits of low and high temperatures.

Problem 3 [25 points]
Find the Fermi energy of a quantum gas of ultra-relativistic fermions with the energy-momentum relation $\varepsilon = cp$, where c is the speed of light.