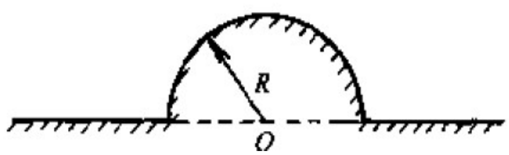
Classical Electrodynamics Qualifying Exam (2.5 hours)
January 18, 2017
Open-book, closed-notes

1. [20 points] A long straight wire runs parallel to an infinite, grounded conducting plane from a distance a above. The wire carries a line charge density λ. Find the force per unit length on the wire, and the surface charge density on the conducting plane.

2. [20 pts] Three charges, $-q$, $2q$ and $-q$ are located on the z-axis at $z=a$, 0 and $-a$ respectively. Compute the electric dipole moment p, the electric quadrupole moments Q_{ij}, and the scalar potential $\Phi(r,\theta,\phi)$ for $r \gg a$.

3. [20 pts] A non-charged conducting hemisphere with radius a is placed on top of an infinite uniformly charged conducting plate (charge density σ). The potential of the conducting plate is V. Find the total induced charge on the hemisphere.



4. [20 pts] Consider a dielectric liquid of permittivity that extends to infinity. There exists a uniform electric field $E_0\hat{z}$ inside the liquid. Now a spherical bubble of radius a is formed inside the liquid. Assume air inside the bubble has permittivity ϵ_0. Write down the formal solution to the scalar potential Φ and boundary conditions needed to solve the potential.

5. [20 pts] Total charge Q is uniformly distributed on a spherical surface of radius R. The sphere is centered at the origin and spins around the z axis with angular velocity ω. Assume magnetic permeability μ_0 everywhere. (a) Show that away from the spherical surface, the magnetic field can be expressed as $\mathbf{H}=-\nabla\Phi_M$, with $\nabla^2\Phi_M=0$. (b) Find the surface current density $\mathbf{K}$ and the boundary conditions for the magnetic induction $\mathbf{B}$ at r = R. (c) Find Φ_M inside and outside of the sphere. (d) Find $\mathbf{B}$ inside the sphere.