QM Qualifier May 2014

Problem 1 (30points)

In a two-state system, two sets of basis kets, $|e\pm\rangle$ (denoted as e-basis) and $|d\pm\rangle$ (d-basis) are related as $|d\pm\rangle = \frac{1}{\sqrt{2}}(|e+\rangle \pm i|e-\rangle)$. The matrix form for a state vector $|\alpha\rangle = \begin{pmatrix} \cos\beta \\ e^{i\varphi}\sin\beta \end{pmatrix}$ is given in d-basis and the operators of two observables are gives in e-basis as, $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) (10pts) Calculate the expectation value for A and B.

(b) (10pts) Calculate the uncertainty for both observables.

(c) (10pts) Verify the uncertainty principle.

Problem 2 (20points)

A state for simple harmonic oscillator of frequency ω starts (at t=0) from an arbitrary superposition of two number states as, $|\alpha,0\rangle = \cos\theta|n\rangle + e^{i\varphi}\sin\theta|l\rangle$, where θ and φ are real and $n > l$.

(a) (5pts) Write the state vector at time t.
(b) (5pts) What is the energy expectation value at time t. Is it a periodic function of time? If yes, what is the period?
(c) (10pts) Calculate the expectation value of potential energy at time t. Is it a periodic function of time? If yes, what is the period?

Problem 3 (30 points)

Let $|jm\rangle$ represent the eigenket of the angular momentum J^2 and J_z of a system. The system is in a state $|\alpha\rangle$, which is a normalized superposition of $|11\rangle$, $|10\rangle$, and $|1-1\rangle$. If you measure J_z, the possible values are $\hbar$, 0, and $-\hbar$ with equal probability.

(a) (10points) Find an explicit form of $|\alpha\rangle$ as the superposition of the eigenket $|jm\rangle$. Write the density matrix.
(b) (20points) Calculate expectation value $\langle J_x\rangle$, $\langle J_y\rangle$, and $\langle J_z\rangle$.

Problem 4 (20points)

Two spin-1/2 particles A and B are set to an entangled state, $|\alpha\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$. $|+-\rangle$ denotes particle A in spin-up state along *z*-direction while particle B in the spin-down state along *z*-direction.

(a) (5 points) When S_z is measured for particle A, what are the possible values and corresponding probability if B is not measured?

(b) (5 points) What results should be for the same question in (a) if particle B is measured at the same time to be in spin-up state?

(c) (10 points) Along direction *x*, which is perpendicular to *z*-axis, S_x of both particles in state $|\alpha\rangle$ are measured. If the result for particle B is $S_{xB} = \frac{\hbar}{2}$, what should be the result for A? Show the steps leading to your answer.