PhD Qualifying Exam: Quantum Mechanics, Fall 2019
(Open book – Sakurai)

1. Evaluate the x-p uncertainty product $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ for the ground state and first excited state of a one-dimensional particle confined between two rigid walls,

$$V = \begin{cases} 0 & , \quad 0 < x < a \\ \infty & , \quad \text{otherwise} \end{cases}$$

2. Consider an electron confined in a potential $V(x) = \frac{1}{2}x^2$ and subjected to an electric field E_0. Write down the ground state energy and wave function of the electron.

3. Consider a free particle of mass m and energy E moving in three dimensions. What is the most general wave function of the particle:
   (a) If it is an eigenstate of energy?
   (b) If the eigenstate is also an eigenstate of momentum?
   (c) If the eigenstate is also an eigenstate of parity?
   (d) If the eigenstate has zero average linear momentum?
   (e) If the particle is in the eigenstate of angular momentum quantum number $l = 2$?

4. Consider a particle in the three-dimensional potential $V(r) = r^2$, where r is the radial distance from the origin.
   (a) Write down the most general wave function of the particle in two different coordinate systems.
   (b) If a particle is described by the wave function

$$\psi(x, y, z) = (x + y + 3z)f(r) \ , \tag{1}$$

      i. Is it an eigenfunction of L^2? If so, what is the l-value (orbital quantum number)? If not, what are the possible values of l that we may obtain when L^2 is measured?
      ii. What are the probabilities for the particle to be found in all possible states with different values of the magnetic quantum number m_l?

5. Consider an electron described by the Hamiltonian H,

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

   (a) At $t = 0$, we measure the energy of the electron. What possible values will we obtain?
   (b) At later time t, we measure the energy again. How is it related to the energy we measured at $t = 0$?
   (c) If at $t = 0$, the electron is equally likely to be in the two possible stationary states, write down its most general state vector at $t = 0$.
   (d) What is the probability that at time $t = 5$ the electron will be in a state different from its initial state?
   (e) Express the Hamiltonian in terms of Pauli matrices and calculate the magnetic field applied on the electron.