Qualifying Exam, Electrodynamics, Aug. 2016

1. [30 pts] A thin spherical shell of radius a is cut along the equator and the upper hemisphere is held at constant scalar potential V while the lower hemisphere is held at $-V$.

(a) Determine the scalar potential $\Phi(r, \theta)$ for $r > a$ and $r < a$, where r is the radius measured from the center of the sphere and θ is the polar angle in spherical coordinates. You may express your answer in terms of constant $c_l = (2l + 1) \int_0^1 P_l(x)dx$, e.g. $c_1 = 3/2$.

(b) Find the total charge and the electric dipole moment of the shell. Hint: you may consider the limit of $r \to \infty$.

2. [20 pts] A capacitor is made of two concentric conducting cylinders of radii a and c ($c > a$) and length $L \gg c$. Between the two cylinders, the region $a < r < b$ is filled with a dielectric material with permittivity ϵ while the remaining volume $b < r < c$ is empty. Suppose the line charge density is $\pm\lambda$ at the cylindrical surface $r = a$ and $r = c$ respectively.

(a) Find the capacitance (per unit length) of the system.

(b) Find the polarization charge per unit length at the surface of the dielectric, $r = b$.

3. [30 pts] Charge Q is uniformly distributed inside a sphere of radius R. The sphere rotates with constant angular velocity ω.

(a) Find the magnetic field $\mathbf{B}$ at the center of the sphere.

(b) Find the vector potential $\mathbf{A}(\mathbf{r})$ in the far field $r \gg R$.

4. [20 pts] A plane electromagnetic wave in vacuum has angular frequency ω. The electric field is polarized along the $\hat{z}$ axis and its maximum amplitude is E_0. At the origin and time $t = 0$, the electric field is measured to be $E_x = E_y = 0$, $E_z = E_0$, and the y component of the magnetic field is found to be zero, $B_y = 0$.

(a) Find $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ in terms of ω, c, and E_0.

(b) The wave is incident normally on a perfectly absorbing flat screen. Find the radiation pressure (force per unit area) on the screen.