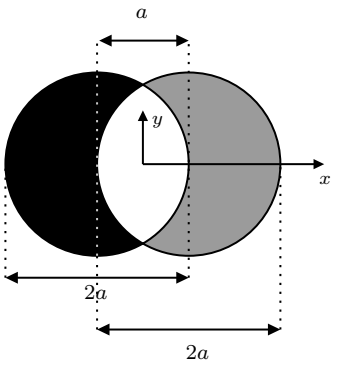
E & M Qualifying Exam

Tuesday, January 17th, 2023

This exam has four problems, each equally weighted.

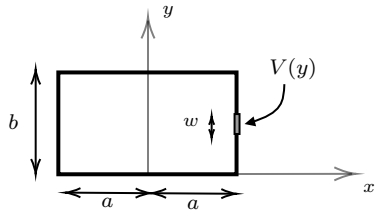
1. Two long and straight conductors each carry a current I in opposing directions; their cross–sections are shown in the figure with black and gray fill. Current flows into the page on the right conductor; current flows out of the page on the left conductor. Assume a permeability of μ_o everywhere.

   Find the magnetic field in (a) the empty space between the conductors and (b) the empty space outside of both conductors.



2. The cross-section of a long and straight rectangular duct with sides of length $2a$ and b are shown in the following figure. The duct is grounded except in a region of width w centered on $y = b/2$. In this region, the potential V has a maximum value of $(b/w)V_o$ at $y = b/2$ and decreases linearly to zero at $y = b/2 - w/2$ and $y = b/2 + w/2$ (so that $V(y)$ has a triangular shape).

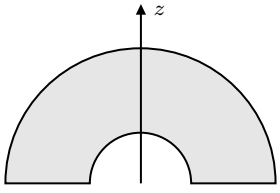
   Find the potential in the duct in the limit that $w/b \to 0$.



3. A spherical capacitor with inner radius a and out radius b is filled with a variable dielectric $\epsilon = \epsilon_0(1 + \epsilon_r \cos^2\theta)$, where θ is the polar angle.

   (a) Find the capacitance. (b) If the inner surface is grounded and the outer surface is held at V_o, find the bound and free charge densities.

4. A thick hemispherical shell of inner radius a and outer radius b has a magnetization of $M_o\hat{\mathbf{z}}$.



   (a) Find the bound current density.
   (b) Find the magnetic field at the origin.
   (c) Describe a procedure for computing the magnetic field for $r > b$.