Qualifying exam - January 2022

Electricity and Magnetism

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [40 points]
A point charge q is a distance $d > R$ away from the center of an electrically neutral conducting sphere.

1. Find the charge density on the surface of the sphere. [20 points]
2. Find the force acting on the charge q. [10 points]
3. Find the potential energy of the system. [10 points]

Problem 2 [20 points]
Calculate the electric quadrupole moment of a uniformly charged ellipsoid with a total charge q and semi-axes of lengths a, b, and c.

Problem 3 [20 points]
Consider a square loop with side a carrying a steady current I (Fig. 1).

1. Calculate the magnetic field on the z axis normal to the loop and passing though its center O. [10 points]
2. Show that at $z \gg a$, this field approaches the field of a magnetic dipole and find the dipole moment. [10 points]

Problem 4 [20 points]
Find the magnetic dipole moment of

1. Thin spherical shell of radius R carrying a uniform surface charge density σ and rotating around its axis with an angular velocity ω. [10 points]
2. Thin disk of radius R carrying a uniform surface charge density σ and rotating around its axis (which is perpendicular to the plane of the disk) with angular velocity ω. [10 points]

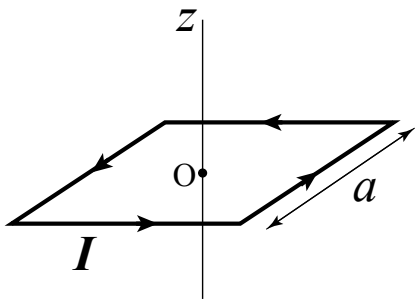

Figure 1: Square loop with current I.