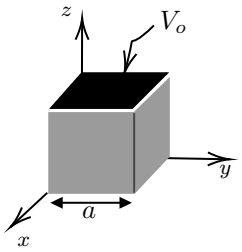
E & M Qualifying Exam

Tuesday, January 19, 2021

This exam has three problems, each worth 33 points.

1. Five sides of a hollow conducting cube with side length a are grounded and one side is at potential V_o. (a) Find the potential at the center of the cube. (b) Starting with Laplace's equation in 3-D Cartesian coordinates, derive a a general equation for the potential $\Phi(x, y, z)$ in Cartesian coordinates. (c) Find $\Phi(x, y, z)$ inside of the cube.



2. A uniform line of charge with charge density λ_o and length $8a$ is formed in the shape of a square with side length $2a$. The square lies in the $x - y$ plane and is centered on the origin.

   Find an expression for the potential on the z-axis for $z < \sqrt{2}a$ in the form $\Phi(z) = A + Bz + Cz^2 + Dz^3$, where the constants $A - D$ depend on one or more of λ_o, a, ϵ_o, and dimensionless integrals (you do not need to evaluate the integrals).

3. The current loop of radius b shown in the figure is in the $z = d$ plane carries a current I. There is an external magnetic field of

$$\mathbf{B}_{ext} = \frac{\mu_o}{4\pi}\frac{m_o}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

   that is due to a magnetic dipole at the origin.

   Compute the force and torque on the loop.