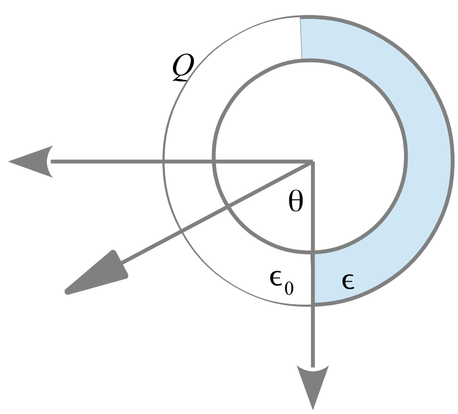
Classical Electrodynamics Qualifying Exam (3 hours)
January 16, 2019
Open-book, closed-notes

Q1. [20 points] The potential on an isolated spherical shell is not equal anywhere. It is vacuum inside and outside the spherical shell. Prove that the potential at the center of the sphere is the average of the potential on the spherical shell.

Q2. [20 pts] An infinitely long cylindrical conductor with radius a is placed in a uniform electrical field $\vec{E}$. The axis of the cylinder is perpendicular to $\vec{E}$. Calculate the induced surface charge density σ of the cylinder.

Q3 [20 pts] Two concentric conducting spheres of inner and outer radii a and b , respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectic (of dielectric constant ϵ/ϵ_0), as shown in the figure.
(a) Find the electric field everywhere between the spheres. (b) Calculate the polarization-charge density induced on the inner surface on the dielectric $r=a$.



Q4. [20 pts] Charge Q is uniformly distributed on a conduction sphere with radius R . The sphere is rotating around its z axis at an angular velocity ω . Suppose the magnetic permeability μ_0 is the same outside and inside the sphere. Solve for the magnetic induction $\mathbf{B}(r)$ using the known quantities Q , R , and ω for both $r>R$ and $r<R$.

Q5. [20 pts] Suppose a magnetostatic field is produced by permanent magnetization $\mathbf{M}(\mathbf{r})$ of a local domain. (a) Show that a scalar potential $\phi(\mathbf{r})$ exists such that $\nabla^2\phi(\mathbf{r})=\nabla\cdot\mathbf{M}(\mathbf{r})$. (b) Prove $\int \mathbf{B}(\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=0$.