E & M Qualifying Exam

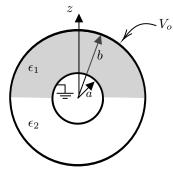
Tuesday, August 16, 2022

This exam has four problems, each equally weighted.

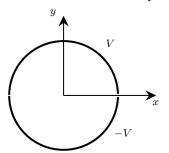
1. A capacitor is formed by two concentric spherical shells of radius a and b that are held at potentials of 0 and V_o , respectively. Half of the space between the shells is filled with a dielectric with permittivity ϵ_1 and the other half with ϵ_2 . A cross-section of the capacitor is shown in the figure.

a. Explain why the radial electric field inside capacitor has no ϕ dependence and no θ dependence, where θ is the angle from z-axis and ϕ is the azimuthal angle.

b. Find the capacitance.



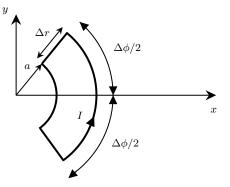
2. A long split cylindrical shell is centered on and parallel to the z-axis. The side with y > 0 is held at a potential of V. The side with y < 0 is held at a potential of -V. An thin insulator at y = 0 separates the two sides. Find the potential inside the cylinder.



3. A wire loop carries a current as shown in the following diagram. At the origin, there is a magnetic dipole with moment $m\hat{\mathbf{z}}$.

a. For $\Delta \phi \ll 1$ and $\Delta r/r \ll 1$, where r is the cylindrical radial distance, find an approximate expression for the net force on the loop.

b. Find an exact expression for the net force on the loop for arbitrary $\Delta\phi$ and $\Delta r.$



4. Two large parallel conducting plates are grounded. Between them there is a uniformly charged slab. Assuming the dimensions of the plates and slab are such that the field is one-dimensional, find the force per unit area on the top plate.

