E & M Qualifying Exam

Fall 2018

August 22, 2018; 1:00 pm - 4:00 pm

This exam has four problems, each worth 25 points.

1. A charged conducting sphere of radius R is embedded in a material that is infinite in extent and has a permittivity of ϵ. The external electric field is uniform and has a magnitude of E_o. The sphere has a net charge of q.

   Find the electric field (a) inside and (b) outside the sphere.

2. A non-conducting spherical shell of radius R is centered at the origin. The surface for $z > 0$ has a net charge q_o that is uniformly distributed. The surface for $z < 0$ has a net charge of $-q_o$ that is uniformly distributed.

   Find an expression for the electrostatic potential in terms of r and the first four Legendre polynomials, $P_l(\theta)$, for $r > R$.

3. An infinitely long wire has a radius R and permeability μ_1 carries a current I with a uniform current density. The wire is placed into a region of space with a uniform field $\mathbf{H}$ with magnitude H_o and direction perpendicular to the axis of the wire.

   Determine $\mathbf{H}$ inside the wire.

4. For electrostatic potentials Ψ and charge densities ρ, the following relationship holds

$$\int_V \rho_2 \Psi_1 \, d^3x = \int_V \rho_1 \Psi_2 \, d^3x$$

   (a) Derive this equation. (b) What is the volume V? (c) Given that ρ_1 corresponds to a charge Q_1 uniformly distributed on a sphere of radius R and ρ_2 corresponds to a charge Q_2 uniformly distributed in the volume of this sphere, show explicitly that the above relationship holds.