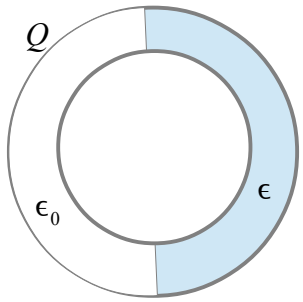
Classical Electrodynamics Qualifying Exam (3 hours)
August 23, 2017
Open-book, closed-notes

1. [20 pts] A charged ring of radius a is placed in parallel to a grounded, infinitely large conducting plane. The distance between the charged ring and the conducting plane is d , and the line charge density on the ring is λ . Write the electrical potential $\Phi(\mathbf{r})$ anywhere in terms of Legendre polynomials.

2. [20 pts] Two concentric conducting spheres of inner and outer radii a and b , respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectic (of dielectric constant ϵ/ϵ_0), as shown in the figure.
(a) Find the electric field everywhere between the spheres. (b) Calculate the polarization-charge density induced on the inner surface on the dielectric $r=a$.



3. [20 pts] Charge Q is uniformly distributed on a conduction sphere with radius R . The sphere is rotating around its z axis at an angular velocity ω . Suppose the magnetic permeability μ_0 is the same outside and inside the sphere. Solve for the magnetic induction $\mathbf{B}(r)$ using the known quantities Q , R , and ω for both $r>R$ and $r<R$.

4. [20 pts] Suppose a magnetostatic field is produced by permanent magnetization $\mathbf{M}(\mathbf{r})$ of a local domain. (a) Show that a scalar potential $\phi(\mathbf{r})$ exists such that $\nabla^2\phi(\mathbf{r})=\nabla\cdot\mathbf{M}(\mathbf{r})$. (b) Prove $\int \mathbf{B}(\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=0$.

5. [20 pts] A thick conducting spherical shell is placed in a uniform electrical field $\vec{E}_0$. The outer and inner radii of the shell are R_1 and R_2 , respectively. An electric dipole (with dipole moment $\vec{p}$) is placed at the center of the sphere. The angle between $\vec{p}$ and $\vec{E}_0$ is α . Suppose the electrical potential of the sphere is V_0 . (1) Find the potential inside and outside the sphere; and (2) Calculate the energy and force of the dipole in the uniform field.