Classical Mechanics Qualifier Exam (14 January 2020)

NAME: G-NUMBER:

(1.) A point particle moves in space under the influence of a force derivable from a generalized potential U of the form:

$$U(\mathbf{r}, \mathbf{v}) = V(r) + \boldsymbol{\beta} \cdot \mathbf{L}, \tag{1}$$

where $\mathbf{r}$ is the radius vector from a fixed point, $\mathbf{L}$ is the angular momentum about that point, and $\boldsymbol{\beta}$ is a fixed vector in space.

(a) State the Lagrangian equation.

(b) Write down the equation for the generalized force Q_j as a function of the generalized potential $U(q, \dot{q})$?

(c) Find the components of the force on the particle in both Cartesian and spherical polar coordinates, on the the basis of the relationship between Q_j and $U(q, \dot{q})$ (the relationship from (b)).

(30 points)

(2.) *(a)* Reverse the Legendre transformation to derive the properties of $L = L(q_i, \dot{q}_i, t)$ from $H = H(q_i, p_i, t)$, treating the $\dot{q}_i$ as independent quantities, and show that it leads to the Lagrangian equations of motion.

(b) By the same procedure find the equations of motion in terms of the function

$$L'(p, \dot{p}, t) = -\dot{p}_i q_i - H(q, p, t) \tag{2}$$

(30 points)

(3.) *Oscillations:* Consider a linear symmetrical triatomic molecule. In the equilibrium condition, two atoms of mass m symmetrically located on each side of an atom of mass M. All three atoms are on straight line, the equilibrium distances apart being d. Consider vibrations only along the line of the molecule. The inter atomic potential can be approximated by two spring of force constant k joining the three atoms. Introduce coordinates relative to the equilibrium position.

(a) Sketch clearly the problem.

(b) Write down the potential and kinetic energies and explain each of them.

(c) Write down the secular equation and determine the eigenvalues. What is the physical meaning of the these eigenvalues?

(20 points)

(4.) The Hamiltonian for a system has the form

$$H = \frac{1}{2}\left(\frac{1}{q^2} + p^2 q^4\right) \tag{3}$$

(a) Find the equation of motion for q.

(b) Find the canonical transformation that reduces H to the form of a harmonic oscillator. Show that the solution for the transformed variables is such that the equation of motion in part a is satisfied.

(20 points)

(100 points in total.)