Classical Mechanics Qualifier Exam (18 August 2020)

NAME: G-NUMBER:

(1.) Derive the Canonical equations of Hamilton using the Legendre transformation for the Hamiltonian.

(30 points)

(2.) A point particle moves in space under the influence of a force derivable from a generalized potential U of the form:

$$U(\mathbf{r}, \mathbf{v}) = V(r) + \boldsymbol{\gamma} \cdot \mathbf{L}, \tag{1}$$

where $\mathbf{r}$ is the radius vector from a fixed point, $\mathbf{L}$ is the angular momentum about that point, and $\boldsymbol{\gamma}$ is a fixed vector in space.

(a) State the Lagrange's equation.

(b) Write down the equation for the generalized force Q_j as a function of the generalized potential $U(q, \dot{q})$?

(c) Find the components of the force on the particle in both Cartesian and spherical polar coordinates, on the the basis of the relationship between Q_j and $U(q, \dot{q})$ (the relationship from (b)).

(d) Show that the components in the two coordinate systems are related to each other as in

$$Q_j = \sum_i \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} \tag{2}$$

(40 points)

(3.) Starting from the principle least action (or Hamilton's principle) derive Lagrange's equations.

(30 points)

(100 points in total.)